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INTERFEROMETRY USING DIRECT AND REFLECTED RAYS FROM A RADIO SOURCE

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Contents

Keywords: Observational astronomy, astronomical methods, radio astronomy, interferometry, radio wave propagation, reflection, WiFi signal, antennas, interference

Prerequisites: Electromagnetic wave propagation, reflection and interference, and antenna basics

Materials:

- **Hardware:**
  a. A transmitter: Smartphone with hot-spot or WiFi router
  b. A receiver: Laptop/smartphone or any other similar portable device that can be connected to WiFi and has provision to show WiFi signal strength
  c. Distance measurement tape/ruler
  d. Metal sheet (or) cardboard and aluminium foil

- **Software:**
  Based on the device used as the receiver, install one of the following software/apps:
  a. Mac OS/ Linux/Unix: Wireshark – Download Wireshark
  b. Windows OS: Vistumbler – Vistumbler - Open Source WiFi scanner and channel scanner for windows
  c. Linux Ubuntu: Python notebook to read and record WiFi signal strength – https://github.com/nrciucaa/WifiSignal
  d. Android OS: WiFi Analyzer – WiFi Analyzer - Apps on Google Play

  The installation process of these software can be found by following the link: Software/Apps/Codes_Wifi_experiments
  We conducted this experiment initially for the Radio Astronomy Winter School 2020 (RAWS2020). The recordings relating to the experiment are available in the following YouTube playlist: Radio Astronomy Winter School 2020 playlist

1 | Introduction

This experiment explores the concept of radio interferometry in the context of a sea-surface interferometer. The sea surface interferometer or the sea cliff interferometer derives its name from the antenna being placed on a cliff along a seashore. The sea surface acts as a reflecting plane, thus resulting in interference fringes appearing at the antenna due to interference between direct and reflected waves from a source. Such interferometers, constructed in Australia, were used in the 1940s-50s to detect radio emissions from the Sun and other compact radio sources. MaCready et al. (1946) reported the first positional association between solar radio bursts and sunspots using such a telescope. It was also the first time a new principle was introduced, that is, the interferometer was sensitive to one of the Fourier components of the source and one could get the brightness distribution of the source if one could measure multiple such Fourier components (MaCready et al., 1947; Sullivan 1991). This experiment laid the foundation for modern radio interferometers. The sea surface interferometer was later used to detect radio emissions from Cygnus A at 100 MHz (Bolton et al. 1953).

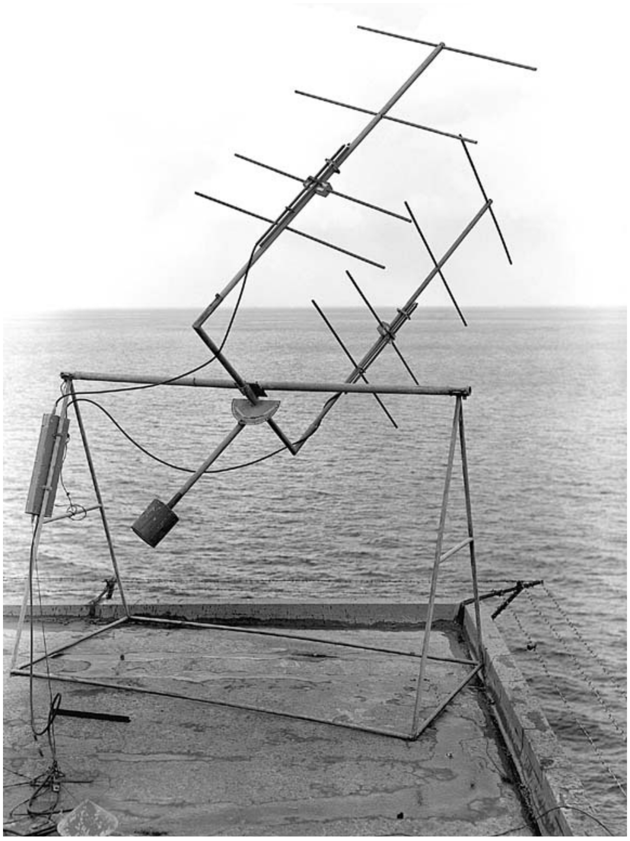

Figure 1.1: Ruby Payne-Scott, Don Yablsey and John Bolton were the first to use these Yagi antennas at Dover Heights, Sydney, Australia, in 1947. This setup was used for solar and compact radio source observations at 100 MHz.

Figure 1.2: Ruby Payne-Scott, Don Yablsey and John Bolton were the first to use these Yagi antennas at Dover Heights, Sydney, Australia, in 1947. This setup was used for solar and compact radio source observations at 100 MHz.

(Source: https://www.atnf.csiro.au/outreach/about/history/doverheights/index.html)

The primary benefit of using an interferometer is achieving higher resolution with two or more small antennas rather than scaling the collecting area of a single antenna radio telescope. The angular resolution of a two-element interferometer is

$$\theta \approx \frac{\lambda}{d} \qquad \ldots(1)$$

where λ is the wavelength and d is the separation between the antennas. The sea surface interferometer was used to resolve the Sun and detect sun spots (McCready et al., 1947). It was also used to resolve several galactic sources for the first time at frequencies in the 100-200 MHz band (see Fig. 3). Resolving these sources would have been impossible with a single dish antenna at that time.

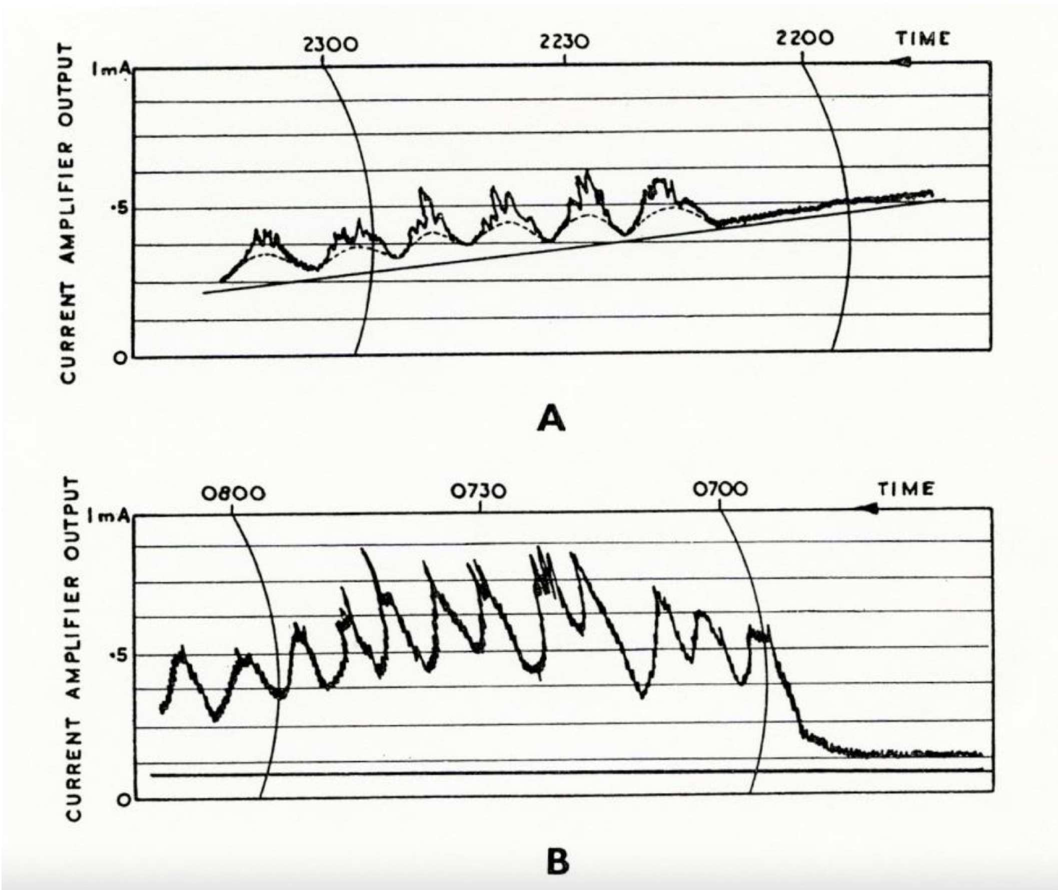

Figure 1.3: A) Interference pattern recorded by the sea surface interferometer recorded at 100 MHz for the Cygnus radio source. B) Interference pattern recorded by the sea surface interferometer for the Sun. (Source: https://core.ac.uk/download/pdf/211503498.pdf)

Such interferometric observations were affected by factors like imperfect reflectivity of the surface, imperfections on the reflecting surface due to sea waves, and the curvature of the Earth (for details of the experiment, see Bolton et al., 1953). These factors will be ignored or found to be not relevant to our experiment.

2 | Revisiting Young's double slit experiment

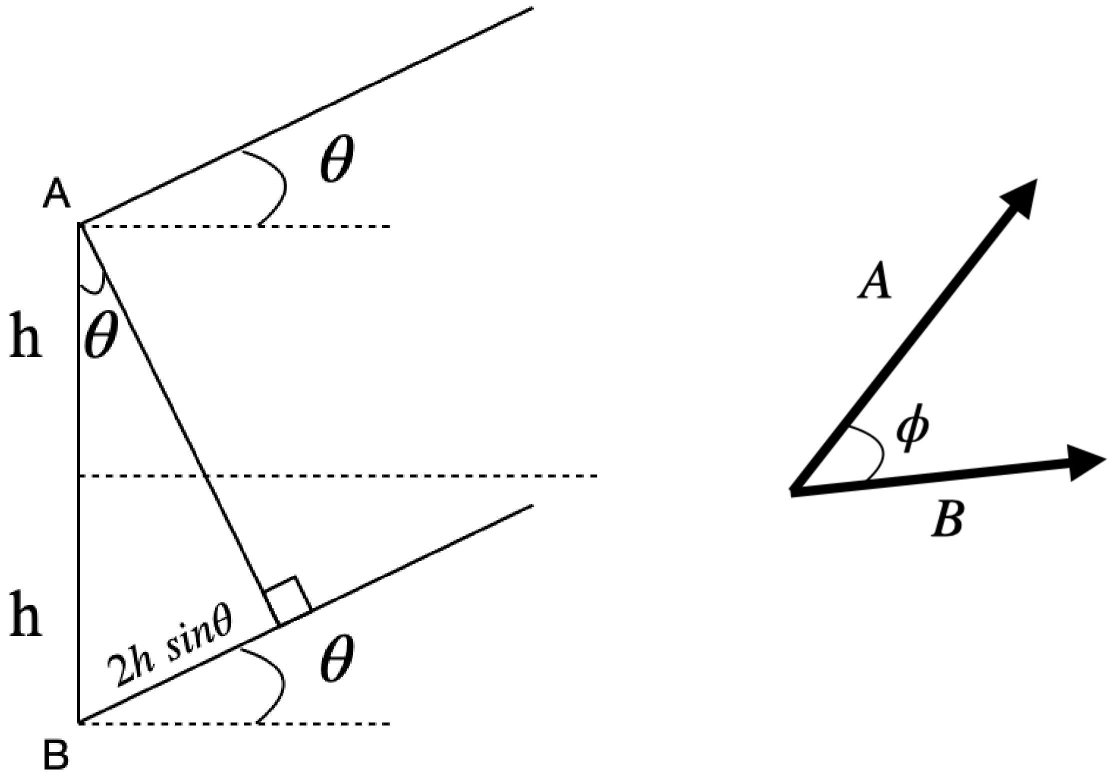

Figure 2.1: A schematic of the sea surface interferometer (left). Phasor representation of signals at A and B (right).

The experiment is analogous to Young's Double Slit experiment with a modification, i.e., the slits are antennas receiving radiation from a distant source. Following the schematic representation in Fig. 4, let us assume two antennas at locations A and B pointed at a source in the sky. The voltages from the two antennae are added to give the receiver output. We can assume the source to be at infinity so that the radiation is received in the form of plane waves at the antenna location. In this case, the wave planes have to travel an extra distance ΔL to antenna B compared to antenna A, which is given as follows.

$$\Delta L = 2 \times h \times \sin(\theta)$$

Here is the altitude of the source in the sky, and '$2h$' is the separation between the two antennas. This additional path travelled by the light creates a phase difference between the voltages at the terminals of the two antennas. We can get the following interference conditions:
Constructive interference,

$$2\,h \sin\theta = m\,\lambda \qquad m=1,2,3... \qquad ...(3)$$

Destructive interference,

$$2\,h \sin\theta = (m+1/2)\,\lambda \qquad m=1,2,3... \qquad ...(4)$$

where, λ is the wavelength of observation and m is the order of the interference fringe. The voltage phasors at the two antennas have a phase difference ϕ between them, as shown in Fig. 3.

$$\phi = \frac{2\pi \; 2h\sin\theta}{\lambda} = \frac{4\pi\, h\sin\theta}{\lambda} \qquad \text{...(5)}$$

If the amplitudes of the two phasors are A and B, then their sum, C, can be defined as follows.

$$C^2 = A^2 + B^2 + 2AB\cos\phi \qquad \text{...(6)}$$

Let P be the power detected at the receiver, then P is proportional to C^2. Now, let $A^2 = B^2 = P_0$. Putting these terms in Eq. 5 gives us,

$$P = 2P_0(1 + \cos\phi) \qquad \text{...(7)}$$

Substituting Eq. 5 here gives the power received as a function of θ.

$$P = 2P_0[1 + \cos(4\pi \sin\theta) \qquad \text{...(8)}$$

3 | The Experiment

In the sea surface interferometer, one antenna is placed on a cliff near a seashore. The sea surface forms a reflecting plane which creates a virtual image of the antenna below the cliff from the viewpoint of the incoming ray. Hence, using just one antenna and an appropriately oriented reflecting surface, one can effectively create an interferometer. In this case, the direct and reflected rays will have the same phase relations and interference patterns as those of a two-antenna setup modelled above.

We will try to set up a similar experiment using a transmitter (mobile phone/router) as the source, a receiver (laptop/mobile phone) and a reflecting surface (aluminium foil). The experimental setup is as shown in Fig. 4.

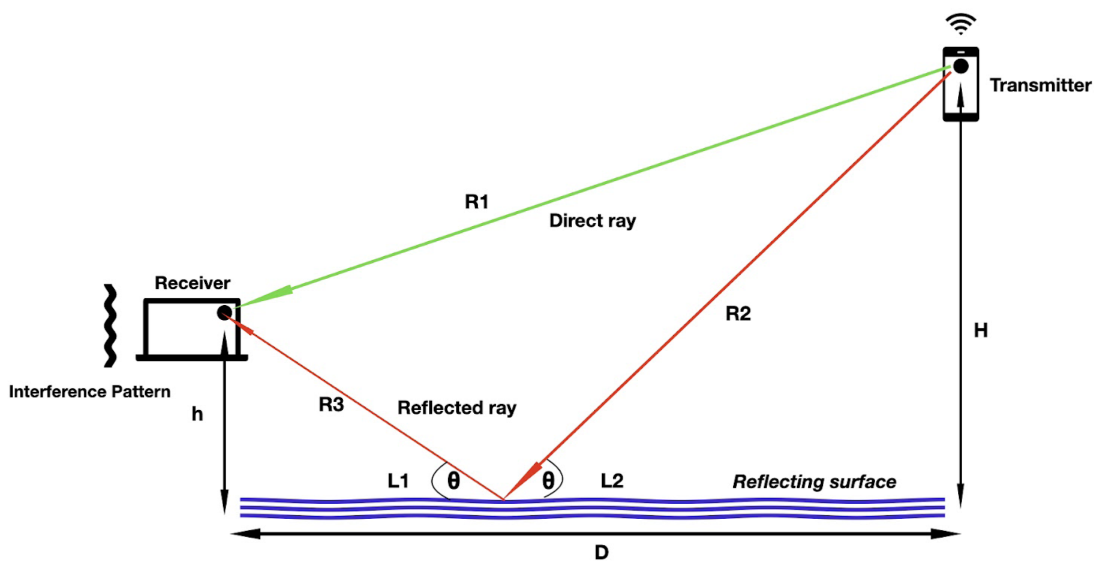

Figure 3.1: A diagram of the experimental setup

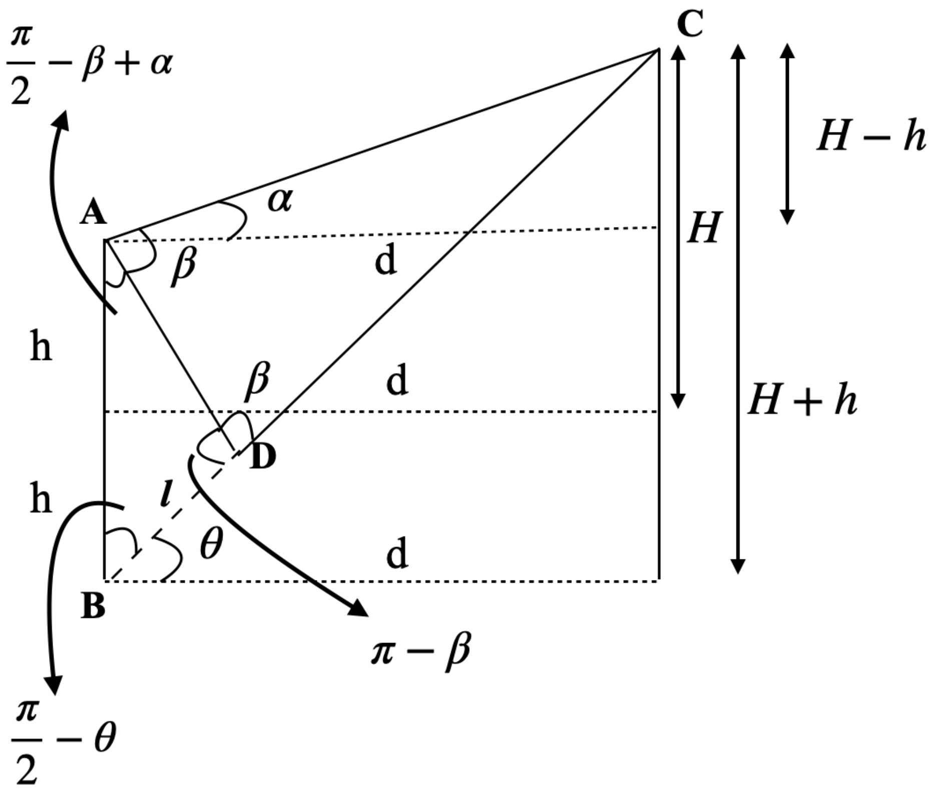

Figure 3.2: A schematic of the experimental setup

There are a few differences between the setups shown in Fig. 4 and Fig. 5 that one needs to keep in mind while doing the experiment. The most crucial difference is that, in our case, the source is close to the antenna. So, one cannot assume the rays directly incident on the receiver and the rays incident on the reflector that eventually interfere to be parallel. Thus, the plane wave approximation of the previous section breaks down here. Following the modified schematic in Fig. 6, antenna A is at a height h from the ground. The ground is covered with aluminium foil to form the reflecting surface. Antenna B is the virtual image antenna formed due to reflection. The source is at C, at a horizontal distance d from our two antennas and at a vertical height H from the reflecting surface. The receiver (antenna A) is at a height h above the reflecting surface.
In the isosceles triangle ACD, $AC = DC$. Therefore, these path lengths do not contribute to any phase difference. To calculate the path difference ℓ = BD, which contributes to the phase difference of the signals received at the two antennas, consider triangle ABD. Using the sine law for triangle ABD, we get the following relationship:

$$\frac{\sin(\pi/2 - \beta + \alpha)}{\ell} = \frac{\sin(\pi - \beta)}{2h} \qquad \ldots(9)$$

Rearranging Eq. 9, we get,

$$\ell = 2h \frac{\sin(\pi/2 - \beta + \alpha)}{\sin(\pi - \beta)} = 2h \frac{\cos(\beta - \alpha)}{\sin\beta} \qquad \ldots(10)$$

The phase can be defined as,

$$\phi = \frac{2\pi}{\lambda} \, l \qquad \ldots(11)$$

Following the arguments from the previous section, we can substitute this phase in Eq. 7.

$$P = 2P_0 [1 + (\cos \phi + \pi)] \qquad \ldots(12)$$

The additional phase of is π because of reflection, which introduces a phase shift of π between the incident and reflected rays. Therefore,

$$P = 2P_0 (1 - \cos \phi) \qquad \ldots(13)$$

α, β and θ are the unknowns in Eq. 10. From Fig. 6, we get the following relations,

$$\tan(\alpha) = \frac{H - h}{d} \qquad \ldots(14)$$

$$\tan(\theta) = \frac{H + h}{d} \qquad \ldots(15)$$

We get β from the sum of angles in triangle ABD.

$$\beta = \frac{\pi - \theta + \alpha}{2} \qquad \ldots(16)$$

See Appendix A for another method to find maxima and minima. Here, *h* is kept fixed, roughly equal to λ , for the duration of the experiment (this is not necessary and can be up to a few multiples of λ). *H* is the height of the mobile phone (moving device) which we change to obtain the interference pattern. The constant distance *d* between the transmitter and receiver planes is maintained at about 1 m. The reflecting surface can be made up of aluminium foil or any metallic material, which is a good reflector of radio waves.

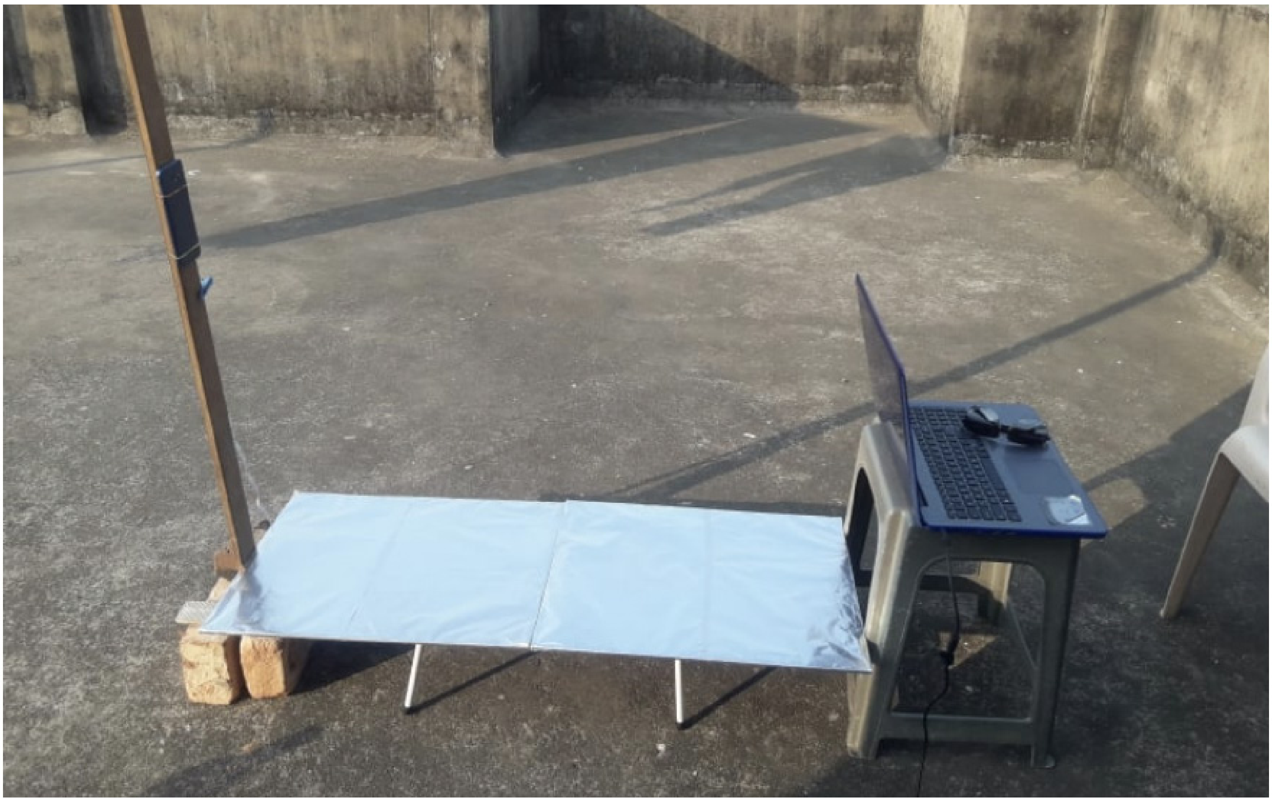

Figure 3.3: An example of our experimental setup which captures the principle of a sea surface interferometer

4 | Experiment setup

1. The line of sight between the transmitter and the receiver must be clear and free of any human/ animal movements across the sightline to avoid any undesired reflection or absorption of the signal.

2. Reduce the possibility of significant reflections (from surrounding walls, ground, roof, furniture, and other objects and structures) that will unavoidably interfere, and contaminate the direct signal. For best results, carry out this experiment in outdoor/open-ground areas or carefully reduce the possible reflections in any indoor setup (and note down the potential sources of contamination). Only direct rays and reflections from the metal surface (aluminium foil) kept on the ground should be allowed to reach the receiver. All other reflecting surfaces should be as far as possible.

3. If using a smartphone as a receiver, turn off its data connection to reduce the power fluctuations due to data transfers. Similarly, keep the smartphone in aeroplane mode to avoid fluctuations in the radiated power due to data transfers and other WiFi activities. For routers, remove the external network connection.

4. The WiFi devices may provide an option to operate at about 2.4 GHz or 5 GHz (each with a few MHz of bandwidth). The central frequency is slightly different for every WiFi device, depending on the band in use. One can select the operating frequency based on the compatibility of the receiving device.

5. Determine the physical location of the WiFi antennas in your devices for accurately measuring distances and ensuring that the corresponding sightline remains clear. The accuracy in distance measurements becomes more critical at smaller distances (in the near field of radiators/receptors) where the signal intensity may deviate from the law that governs its far-field behaviour.

6. Finally, equip the receiver with the appropriate software/application to measure the received WiFi power by following the details/links given under the 'software' section above. The software will enable viewing the signal strength in dBm and transmitter device information such as the operating frequency and bandwidth. Power expressed in dBm is ten times the log (to the base 10) of the power P in milli-watts, PdBm = 10 log(P).

7. Depending on the location of the WiFi antenna in the laptop, its orientation should be adjusted so that there is little or no reflection from the laptop body itself. The orientation can be as shown in Fig. 5, i.e. perpendicular to the plane of rays or as shown in Fig. 7, i.e. the laptop screen facing away from the mobile phone. The screen facing towards the mobile phone is discouraged as the laptop body can act as a secondary reflector to the waves and affect the reading.

5 | Procedure

1. Ensure your router is on, or create a WiFi hotspot on the mobile phone.

2. Connect your receiving device (laptop/smartphone) to the hotspot/WiFi.

3. Disconnect the mobile phone or router from data/internet. Put the mobile phone in Flight Mode or disconnect router network input.

4. Place the laptop (receiver) on a non-metallic base at a height of the order of two above the reflecting surface.

5. Make sure that the reflecting surface is as flat and horizontal as possible.

6. Set the distance between the transmitter and receiver (d) to about 1 meter.

7. Start moving the mobile phone (transmitter) vertically upwards while keeping the distance d constant. Note down the height of the mobile phone and the corresponding received power in a table.

8. Plot the signal strength vs height. Check the graph and compare the positions of maxima and minima with the expected analytical values.

9. Interpret the results.

6 | Tips

1. Keep your mobile phone and laptop charged.

2. The WiFi power detector device might take a few seconds to settle down on a value. Please do not disturb the receiving setup while the reading settles and allow sufficient time for the reading to settle down to its minimum fluctuation level.

3. Depending on the device, the lowest signal strength that can be measured reliably could be between -80 dBm to -95 dBm; hence avoid measurements below -80 dBm (limiting the maximum distance).

4. For plotting, pick any tool/programming language such as Python, Matlab, Origin, Excel, etc.

7 | Observations

Sr. No.	Height	Power (dBm)
1	1 cm	-35 dBm
2	2 cm	-32 dBm
3		
4		
.		

8 | Results

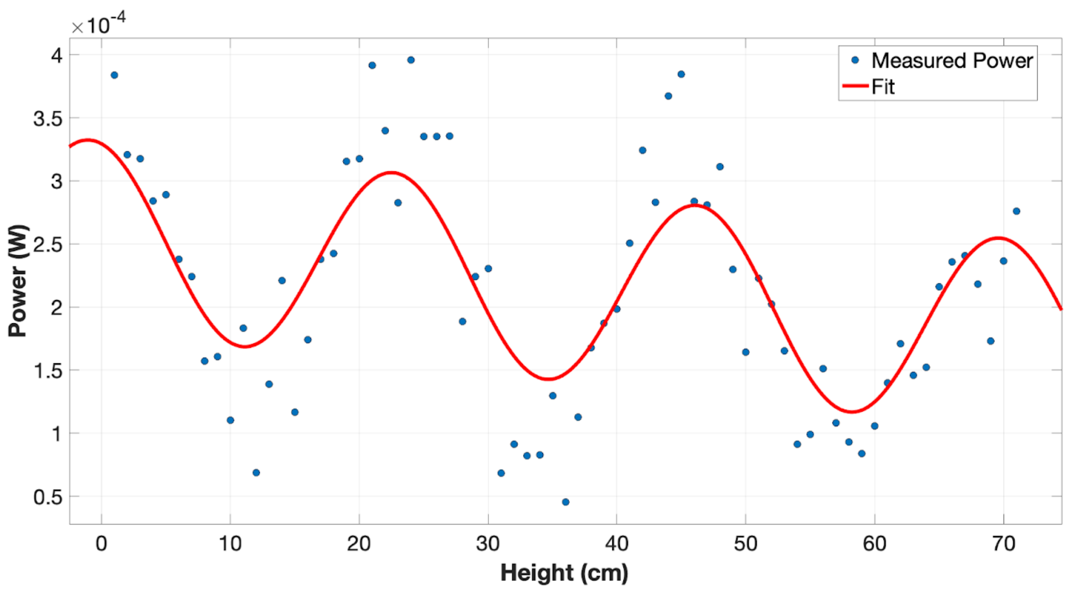

Figure 8.1: Example of data and fit plots. Model variation as a sinusoid is only indicative. The exact form of the variation will depend on the details of the spatial parameters in the experiment. In any case, the peaks will correspond to constructive interference, and the lows will correspond to destructive interference

Fig. 6 shows the results of the experiment. The data are fit with the model expression from Eq. 13. While fitting, it should be noted that due to the finite size of the transmitting device, i.e. the mobile phone, the data plot may have a slight offset along the horizontal axis. Since the distance between the transmitter and the receiver is relatively small, the distance travelled by the direct and the reflected waves will differ and change with height. This difference in path lengths will create different amplitudes for the two waves. Eq. 6 takes into account this effect. One can try to estimate this difference in amplitudes.

9 | References

1. L. L. McCready, Joseph Lade Pawsey and Ruby Payne-Scott "Solar radiation at radio frequencies and its relation to sunspots" Proceedings of the Royal Society of London. Series A. Mathematical and Physical Sciences: Vol 190, No 1022 12 August 1947. http://doi.org/10.1098/rspa.1947.0081

2. Bolton, J. G., Slee, O. B. (1953) Galactic Radiation at Radio Frequencies. V. The Sea Interferometer. Australian Journal of Physics 6, 420-433. 10.1071/PH530420

3. Sullivan, W. T., "Some highlights of Interferometry in early Radio Astronomy", IAU Colloq. 131: Radio Interferometry. Theory, Techniques, and Applications, 1991, vol. 19, p. 132.

4. Bolton, J., Stanley, G., "Variable Source of Radio Frequency Radiation in the Constellation of Cygnus." Nature 161, 312–313 (1948). https://doi.org/10.1038/161312b0

5. A. Richard Thompson, James M. Moran, George W. Swenson Jr, Interferometry and Synthesis in Radio Astronomy, Springer Open A&A Library, pp 19-21

10 | Acknowledgements

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11 | Appendix A:

Consider Fig. 4 for the following equations.
Given: h, D, H (measurement)
Derivation of path difference:

$$Path\,1 = R\,1 \quad \ldots(17)$$

$$Path\,2 = R\,2 + R\,3 \quad \ldots(18)$$

$$R1^2 = d^2 + (H - h)^2 \quad \ldots(19)$$

$$R2^2 = L2^2 + H^2 \quad \ldots(20)$$

$$R3^2 = L1^2 + h^2 \quad \ldots(21)$$

$$d = L1 + L2 \quad \ldots(22)$$

$$\tan\theta = \frac{h}{L1} = \frac{H}{L2} \quad \ldots(23)$$

Therefore,

$$L1/L2 = h/H \quad \ldots(24)$$

Using Eq. 9 and Eq. 11

$$\mathrm{L2} = \frac{d \times \mathrm{H}}{h + \mathrm{H}} \quad \ldots(25)$$

$$\mathrm{L1} = d - \frac{d \times H}{h + H} \quad \ldots(26)$$

From Eq. 12 and Eq. 13

$$R3 = \sqrt{\left(d - \frac{d \times H}{h + H}\right)^2 + h^2} \quad \ldots(27)$$

$$R2 = \sqrt{\left(\frac{d \times H}{h + H}\right)^2 + H^2} \quad \ldots(28)$$

So, the net path difference is,

$$R2 + R3 - R1 \quad \ldots(29)$$

We get the following conditions considering the λ/2 phase shift due to reflection.
For constructive interference

$$R2 + R3 - R1 = (m + 1/2) \times \lambda \quad \ldots.(30)$$

For destructive interference

$$R2 + R3 - R1 = m \times \lambda \quad \ldots.(31)$$

m=0,1,2,3....m